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SECTION 2

FORECASTING OF TRAFFIC

Recommendation E.506

FORECASTING INTERNATIONAL TRAFFIC

1 Introduction

This Recommendation is the first in a series of three Recommendations that cover international telecommunications forecasting.

In the operation and administration of the international telephone network, proper and successful development depends to a large degree upon estimates for the future. Accordingly, for the planning of equipment and circuit provision and of telephone plant investments, it is necessary that Administrations forecast the traffic which the network will carry. In view of the heavy capital investments in the international network, the economic importance of the most reliable forecast is evident.

The purpose of this Recommendation is to give guidance on some of the prerequisites for forecasting international telecommunications traffic. Base data, not only traffic and call data but also economic, social and demographic data, are of vital importance for forecasting. These data series may be incomplete; strategies are recommended for dealing with missing data. Different forecasting approaches are presented including direct and composite methods, matrix forecasting, and top down and bottom up procedures.

Recommendation E.507 provides guidelines for building forecasting models and contains an overview of various forecasting techniques. Recommendation E.508 covers the forecasting of new international telecommunications services.

2 Base data for forecasting

An output of the international traffic forecasting process is the estimated number of circuits required for each period in the forecast horizon. To obtain these values, traffic engineering techniques are applied to forecast Erlangs, a measure of traffic. Figure 1/E.506 outlines two different approaches for determining forecasted Erlangs.

The two different strategies for forecasting are the direct strategy and the composite strategy. The first step in either process is to collect raw data. These raw data, perhaps adjusted, will be the base data used to generate the traffic forecasts. Base data may be hourly, daily, monthly, quarterly, or annual. Most Administrations use monthly accounting data for forecasting purposes.

With the direct strategy, the traffic carried in Erlangs, or measured usage, for each relation would be regarded as the base data in forecasting traffic growth. These data may be adjusted to account for such occurrences as regeneration (see Recommendation E.500).

The old Recommendation E.506 which appeared in the *Red Book* was split into two Recommendations, revised E.506 and new E.507 and considerable new material was added to both.

Figure 1/E.506, p. 1

In both strategies (direct and composite) it is necessary to convert the carried traffic into offered traffic Erlangs. The conversion formula can be found in Recommendation E.501 for the direct strategy and in this Recommendation for the composite strategy.

Composite forecasting uses historical international accounting data of monthly paid minute traffic as the base data. The data may be adjusted by a number of factors, either before or after the forecasting process, that are used for converting paid minutes on the basis of the accounting data into busy-hour Erlang forecasts.

As seen in Figure 1/E.506, the forecasting process is common to both the direct and composite strategy. However, the actual methods or models used in the process vary. Forecasts can be generated, for example, using traffic matrix methods (see § 4), econometric models or autoregressive models (see § 3, Recommendation E.507). There are various other data that are input to the forecasting process. Examples of these are explanatory variables, market segmentation information and price elasticities.

Wherever possible, both the direct and composite forecasting strategies should be used and compared. This comparison may reveal irregularities not evident from the use of only one method. Where these are significant, in particular in the case of the busy hour, the causes for the differences should be identified before the resulting forecast is adopted.

In econometric modelling especially, explanatory variables are used to forecast international traffic. Some of the most important of these variables are the following:

— exports,

— imports,

— degree of automation,

— quality of service,

— time differences between countries,

— tariffs,

— consumer price index, and

— gross national product.

Other explanatory variables, such as foreign business travellers and nationals living in other countries, may also be important to consider. It is recommended that data bases for explanatory variables should be as comprehensive as possible to provide more information to the forecasting process.

Forecasts may be based on market segmentation. Base data may be segmented, for example, along regional lines, by business, non-business, or by type of service. Price elasticities should also be examined, if possible, to quantify the impact of tariffs on the forecasting data.

3 Composite strategy — Conversion method

The monthly paid-minutes traffic is converted to busy-hour Erlangs for dimensioning purposes by the application of a number of traffic related conversion factors for each service category. The conversion is carried out in accordance with the formula:

$$A = Mdh/60e \tag{3-1}$$

where

A is the estimated mean traffic in the busy hour,

M is the monthly paid-minutes,

d is day-to-month ratio,

h is the busy hour-to-day ratio, and

e is the efficiency factor.

The formula is described in detail in Annex A.

4 Procedures for traffic matrix forecasting

4.1 *Introduction*

To use traffic matrix or point-to-point forecasts the following procedures may be used:

— Direct point-to-point forecasts,

— Kruithof's method,

— Extension of Kruithof's method,

— Weighted least squares method.

It is also possible to develop a Kalman Filter model for point-to-point traffic taking into account the aggregated forecasts. Tu and Pack describe such a model in [16].

The forecasting procedures can be used to produce forecasts of internal traffic within groups of countries, for example, the Nordic countries. Another application is to produce forecasts for national traffic on various levels.

4.2 *Direct point-to-point forecasts*

It is possible to produce better forecasts for accumulated traffic than forecast of traffic on a lower level.

Hence, forecasts of outgoing traffic (row sum) or incoming traffic (column sum) between one country and a group of countries will give a relatively higher precision than the separate forecasts between countries.

In this situation it is possible to adjust the individual forecasts by taking into account the aggregated forecasts.

On the other hand, if the forecasts of the different elements in the traffic matrix turn out to be as good as the accumulated forecasts, then it is not necessary to adjust the forecasts.

Evaluation of the relative precision of forecasts may be carried out by comparing the ratios $s(X)/X$ where X is the forecast and $s(X)$ the estimated forecast error.

4.3 *Kruithof's method*

Kruithof's method [11] is well known. The method uses the last known traffic matrix and forecasts of the row and column sum to make forecasts of the traffic matrix. This is carried out by an efficient iteration procedure.

Kruithof's method does not take into account the change over time in the point-to-point traffic. Because Kruithof's method only uses the last known traffic matrix, information on the previous traffic matrices does not contribute to the forecasts. This would be disadvantageous. Especially when the growth of the distinct point-to-point traffic varies. Also when the traffic matrices reflect seasonal data, Kruithof's method may give poor forecasts.

4.4 *Extension of Kruithof's method*

The traditional Kruithof's method is a projection of the traffic based on the last known traffic matrix and forecasts of the row and column sums.

It is possible to extend Kruithof's method by taking into account not only forecasts of the row and column but also forecasts of point-to-point traffic. Kruithof's method is then used to adjust the point-to-point traffic forecasts to obtain consistency with the forecasts of row and column sums.

The extended Kruithof's method is superior to the traditional Kruithof's method and is therefore recommended.

4.5 *Weighted least squares method*

Weighted least squares method is again an extension of the last method. Let $\{C_{ij}\}$ $\{C_{i\cdot}\}$ and $\{C_{\cdot j}\}$ be forecasts of point-to-point traffic, row sums and column sums respectively.

The extended Kruithof's method assumes that the row and column sums are ''true'' and adjust $\{C_{ij}\}$ to obtain consistency.

The weighted least squares method [2] is based on the assumption that both the point-to-point forecasts and the row and column sum forecasts are uncertain. A reasonable way to solve the problem is to give the various forecasts different weights.

Let the weighted least squares forecasts be $\{D_{ij}\}$. The square sum Q is defined by:

$$Q = \sum_{ij} a_{ij}(C_{ij} - D_{ij})^2 + \sum_i b_i (C_{i\cdot} - D_{i\cdot})^2 + \sum_j c_j (C_{\cdot j} - D_{\cdot j})^2 \tag{4-1}$$

where $\{a_{ij}\}$ $\{b_i\}$ $\{c_j\}$ are chosen constants or weights.

The weighted least squares forecast is found by:

in Q BOCAD D fIn $_j$ 15(D fIi $_j$)

subject to $D_{i\cdot} = \sum_j D_{ij}$ $i = 1, 2, . | |$ (4-2)

and

$D_{\cdot j} = \sum_i D_{ij}$ $j = 1, 2, . | |$

A natural choice of weights is the inverse of the variance of the forecasts. One way to find an estimate of the standard deviation of the forecasts is to perform ex-post forecasting and then calculate the root mean square error.

The properties of this method are analyzed in [14].

5 Top down and bottom up methods

5.1 *Choice of model*

The object is to produce forecasts for the traffic between countries. For this to be a sensible procedure, it is necessary that the traffic between the countries should not be too small, so that the forecasts may be accurate. A method of this type is usually denoted as ‘‘bottom up’’.

Alternatively, when there is a small amount of traffic between the countries in question, it is better to start out with forecasting the traffic for a larger group of countries. These forecasts are often used as a basis for forecasts for the traffic to each country. This is done by a correction procedure to be described in more detail below. Methods of this type are called ‘‘top down’’. The following comments concern the preference of one method over another.

Let σ_T^2 be the variance of the aggregated forecast, and σ_i^2 be the variance of the local forecast No. i and $\gamma_{i\backslash dj}$ be the covariance of the local forecast No. i and j . If the following inequality is true:

then, in general, it is not recommended to use the bottom up method, but to use the top down method.

In many situations it is possible to use a more advanced forecasting model on the aggregated level. Also, the data on an aggregated level may be more consistent and less influenced by stochastic changes compared to data on a lower level. Hence, in most cases the inequality stated above will be satisfied for small countries.

5.2 *Bottom up method*

As outlined in § 5.1 the bottom up method is defined as a procedure for making separate forecasts of the traffic between different countries directly. If the inequality given in § 5.1 is not satisfied, which may be the case for large countries, it is sufficient to use the bottom up method. Hence, one of the forecasting models mentioned in Recommendation E.507 can be used to produce traffic forecasts for different countries.

5.3 *Top down procedure*

In most cases the top down procedure is recommended for producing forecasts of international traffic for a small country. In Annex D a detailed example of such a forecasting procedure is given.

The first step in the procedure is to find a forecasting model on the aggregated level, which may be a rather sophisticated model. Let X_T be the traffic forecasts on the aggregated level and σ_T the estimated standard deviation of the forecasts.

The next step is to develop separate forecasting models of traffic to different countries. Let X_i be the traffic forecast to the i^{th} country and σ_i the standard deviation. Now, the separate forecasts $[X_i]$ have to be corrected by taking into account the aggregated forecasts X_T. We know that in general

Let the corrections of $[X_i]$ be $[X'_i]$, and the corrected aggregated forecast then be $X'_T = \sigma'' X'_i$.

The procedure for finding $[X'_i]$ is described in Annex C.

6 Forecasting methods when observations are missing

6.1 *Introduction*

Most forecasting models are based on equally spaced time series. If one observation or a set of observations are missing, it is necessary either to use an estimate of missing observations and then use the forecasting model or to modify the forecasting model.

All smoothing models are applied on equally spaced observations. Also autoregressive integrated moving average (ARIMA)-models operate on equally spaced time series, while regression models work on irregularly spaced observations without modifications.

In the literature it is shown that most forecasting methods can be formulated as dynamic linear models (DLM). The Kalman Filter is a linear method to estimate states in a time series which is modelled as a dynamic linear model. The Kalman Filter introduces a recursive procedure to calculate the forecasts in a DLM which is optimal in the sense of minimizing the mean squared one step ahead forecast error. The Kalman Filter also gives an optimal solution in the case of missing data.

6.2 *Adjustment procedure based on comparable observations*

In situations when some observations are missing, it may be possible to use related data for estimating the missing observations. For instance, if measurements are carried out on a set of trunk groups in the same area, then the traffic measurements on various trunk groups are correlated, which means that traffic measurements on a given trunk group to a certain degree explain traffic measurements on other trunk groups.

When there is high correlation between two time series of traffic measurements, the relative change in level and trend will be of the same size.

Suppose that a time series x_t of equidistant observations from 1 to *n* | as an inside gap | (mu | fIx$_t$ is, for instance, the yearly increase. The gap consists of *k* missing observations between *r* and *r* + *k* + 1.

A procedure for estimating the missing observations is given by the following steps:

i) Examine similar time series to the series with missing observations and calculate the cross correlation.

ii) Identify time series with high cross correlation at lag zero.

iii) Calculate the growth factor ?63$_{r\backslash d+i}$ between *r* and *r* + *k* of the similar time series y_t:

iv) Estimates of the missing observations are then given by:

$$\hat{x}_{r\backslash d+i} = x_r + ?63_{r\backslash d+i}(x_{r+k+1} - x_r)$$

$$i = 1, 2, . | | k$$

(6-2)

Example

Suppose we want to forecast the time series x_t. The series is observed from 1 to 10, but the observations at time 6, 7 and 8 are missing. However a related time series y_t is measured. The measurements are given in Table 1/506.

H.T. [T1.506]

TABLE 1/E.506

Measurements of two related time series; one with missing observations

t	1	2	3	4	5	6	7	8	9	10
x	*100*	*112*	*125*	*140*	*152*	—	—	—	*206*	*221*
y	*300*	*338*	*380*	*422*	*460*	*496*	*532*	*574*	*622*	*670*

Table 1/E.506 [T1.506], p.

The last known observation of x_t before the gap at time 5 is 152, while the first known observation after the gap at time 9 is 206.

Hence $r = 5$ and $k = 3$. The calculation gives:

6.3 *Modification of forecasting models*

The other possibility for handling missing observations is to extend the forecasting models with specific procedures. When observations are missing, a modified procedure, instead of the ordinary forecasting model, is used to estimate the traffic.

To illustrate such a procedure we look at simple exponential smoothing. The simple exponential smoothing model is expressed by:

$$m_t = (1 - a) y_t + a m_{t-1} \tag{6-3}$$

where

y_t is the measured traffic at time t

m_t is the estimated level at time t

a is the discount factor [and $(1 - a)$ is the smoothing parameter].

Equation (6-3) is a recursive formula. The recursion starts at time 1 and ends at n if no observation is missing. Then a one step ahead forecast is given by:

$$\hat{y}_t(1) = m_t \tag{6-4}$$

If some observations lying in between 1 and n are missing, then it is necessary to modify the recursion procedure. Suppose now that $y_1, y_2, \ldots, y_r, y_{r+k+1}, y_{r+k+2}, \ldots, y_n$ are known and $y_{r+1}, y_{r+2}, y_{r+k}$ are unknown. Then the time series has a gap consisting of k missing observations.

The following modified forecasting model for simple exponential smoothing is proposed in Aldrin [2].

where

$$a_k = \frac{a}{1 + k(1 - a)^2} \tag{6-6}$$

By using the (6-5) and (6-6) it is possible to skip the recursive procedure in the gap between r and $r + k + 1$.

In Aldrin [2] similar procedures are proposed for the following forecasting models:

— Holt’s method,

— Double exponential smoothing,

— Discounted least squares method with level and trend,

— Holt-Winters seasonal methods.

Wright [17] and [18] also suggests specific procedures to modify the smoothing models when observations are missing.

As mentioned in the first paragraph, regression models are invariant of missing observations. When using the least squares method, all observations are given the same weight. Hence, missing observations do not affect the estimation procedure and forecast are made in the usual way.

On the other hand it is necessary to modify ARIMA models when observations are missing. In the literature several procedures are suggested in the presence of missing data. The basic idea is to formulate the ARIMA model as a dynamic linear model. Then the likelihood function is easy to obtain and the parameters in the model can be estimated recursively. References to work on this field are Jones [9] and [10], Harvey and Pierse [8], Ansley and Kohn [3] and Aldrin [2].

State space models or dynamic linear models and the Kalman Filter are a large class of models. Smoothing models, ARIMA models and regression models may be formulated as dynamic linear models. This is shown, for instance, in Abraham and Ledolter [1]. Using dynamic linear models and the Kalman Filter the parameters in the model are estimated in a recursive way. The description is given, for instance, in Harrison and Stevens [7], Pack and Whitaker [13], Moreland [12], Szelag [15] and Chemouil and Garnier [6].

In Jones [9] and [10], Barham and Dunstan [4], Harvey and Pierse [8], Aldrin [2] and Bølviken [5] it is shown how the dynamic linear models and the Kalman Filter handle missing observations.

ANNEX A

(to Recommendation E.506)

Composite strategy

A.1 *Introduction*

This annex describes a method for estimating international traffic based on monthly paid-minutes and a number of conversion factors. It demonstrates the method by examining the factors and showing their utility.

The method is seen to have two main features:

1) Monthly paid-minutes exchanged continuously between Administrations for accounting purposes provide a large and continuous volume of data.

2) Traffic conversion factors are relatively stable, when compared with traffic growth and change slowly since they are governed by customers’ habits and network performance. By separately considering the paid minutes and the traffic conversion factors, we gain an insight into the nature of traffic growth which cannot be obtained by circuit occupancy measurements alone. Because of the stability of the conversion factors, these may be measured using relatively small samples, thus contributing to the economy of the procedure.

A.2 *Basic procedure*

A.2.1 *General*

The composite strategy is carried out for each stream, for each direction and generally for each service category.

The estimated mean offered busy-hour traffic (in Erlangs) is derived from the monthly paid-minutes using the formula:

$$A = Mdh/60e \quad \text{(A-1)}$$

where

A is the estimated mean traffic in Erlangs offered in the busy hour,

M is the total monthly paid-minutes,

d is the day/month ratio, i.e. the ratio of average weekday paid-time to monthly paid-time,

h is the busy-hour/day ratio, i.e. the ratio of the busy-hour paid-time to the average daily paid-time,

e is the efficiency factor, i.e. the ratio of busy-hour paid-time to busy-hour occupied-time.

A.2.2 *Monthly paid-minutes (M)*

The starting point for the composite strategy is paid minutes. Sudden changes in subscriber demand, for example, resulting from improvements in transmission quality, have a time constant of the order of several months, and on this basis paid minutes accumulated over monthly intervals appear to be optimum in terms of monitoring traffic growth. A longer period (e.g. annually) tends to mask significant changes, whereas a shorter period (e.g. daily) not only increases the amount of data, but also increases the magnitude of

fluctuations from one period to the next. A further advantage of the one-month period is that monthly paid-minute figures are exchanged between Administrations for accounting purposes and consequently historical records covering many years are normally readily available.

It should be recognized, however, that accounting information exchanges between Administrations often take place after the event, and it may take some time to reach full adjustments (e.g. collect call traffic).

A.2.3 *Day/month ratio (d)*

This ratio is related to the amount of traffic carried on a typical weekday compared with the total amount of traffic carried in a month.

As the number of weekdays and non-weekdays (weekends and holidays) varies month by month, it is not convenient to refer to a typical month, but it should be possible to compute the ratio for the month for which the busy hour traffic is relevant.

Hence if:

X denotes the number of weekdays in the related month

Y denotes the number of non-weekdays (weekend days and holidays) in the selected month, then

[Formula deleted]

(A-2)

where

$$r = \frac{verage\ \ non\text{-}weekday\ \ traffic}{verage\ \ weekday\ \ traffic}$$

The relative amount of non-weekday traffic is very sensitive to the relative amount of social contact between origin and

In a situation where only yearly paid-minutes are available, this may be converted to *M* by a suitable factor.

destination. (Social calls, are, in general, made more frequently on weekends.) Since changes in such social contact would be very slow, r or d are expected to be the most stable conversion factors, which in general vary only within relatively narrow limits. However, tariff policies such as reduced weekend rates can have a significant effect on r and d .

When r is in the region of 1, the Sunday traffic may exceed the typical weekday level. If this is the case, consideration should be given to dimensioning the route to cater for the additional weekend (Sunday) traffic or adopting a suitable overflow routing arrangement.

A.2.4 *Busy-hour/day ratio (h)*

The relative amount of average weekday traffic in the busy hour primarily depends on the difference between the local time at origin and destination. Moderately successful attempts have been made to predict the diurnal distribution of traffic based on this information together with supposed ‘‘degree of convenience’’ at origin and destination. However, sufficient discrepancies exist to warrant measuring the diurnal distribution, from which the busy-hour/day ratio may be calculated.

Where measurement data is not available, a good starting point is Recommendation E.523. From the theoretical distributions found in Recommendation E.523, one finds variations in the busy-hour/day ratio from 10% for 0 to 2 hours time difference and up to 13.5% for 7 hours time difference.

As described above, the composite strategy is implemented as an accounting-based procedure. However, it may be more practical for some Administrations to measure d and h based on occupied time, derived from available call recording equipment.

A.2.5 *Efficiency factor (e)*

The efficiency factor (ratio of busy-hour paid time to busy-hour occupied time, e) converts the paid time into a measure of total circuit occupancy. It is therefore necessary to include all occupied circuit time in the measurement of this ratio, and not merely circuit time taken up in establishing paid calls. For example, the measurement of total circuit occupied time should include the occupied time for paid calls (time from circuit seizure to circuit clearance) and, in addition, the occupied time for directory inquiry calls, test calls, service calls, ineffective attempts and other classes of unpaid traffic handled during the busy hour.

There is a tendency for the efficiency to change with time. In this regard, efficiency is mainly a function of operating method (manual, semi-automatic, international subscriber dialling), the B-subscriber’s availability, and the quality of the distant network.

Forecasts of the efficiency can be made on the basis of extrapolation of past trends together with adjustments for planned improvements.

The detailed consideration of efficiency is also an advantage from an operational viewpoint in that it may be possible to identify improvements that may be made, and quantify the benefits deriving from such improvements.

It should be noted that the practical limit for e is generally about 0.8 to 0.9 for automatic working.

A.2.6 *Mean offered busy hour traffic (A)*

It should be noted that A | s the mean offered busy-hour traffic expressed in Erlangs.

A.2.7 *Use of composite strategy*

In the case of countries with lower traffic volumes and manual operation, the paid-time factors (d and h) would be available from analysis of call vouchers (dockets). For derivation of the efficiency e , the manual operator would have to log the busy-hour occupied time as well as the paid time during the sampling period.

In countries using stored-program controlled exchanges with associated manual assistance positions, computer analysis may aid the composite forecasting procedure.

One consequence of the procedure is that the factors d | nd h | ive a picture of subscriber behaviour, in that unpaid time (inquiry calls, test calls, service calls, etc.) are not included in the measurement of these factors. The importance of deriving the efficiency, e , during the busy hour, should also be emphasized.

ANNEX B

(to Recommendation E.506)

Example using weighted least squares method

B.1 *Telex data*

The telex traffic between the following countries has been analyzed:

— Germany (D)

— Denmark (DNK)

— USA (USA)

— Finland (FIN)

— Norway (NOR)

— Sweden (S)

The data consists of yearly observations from 1973 to 1984 [19].

B.2 *Forecasting*

Before using the weighted least squares method, separate forecasts for the traffic matrix have to be made. In this example a simple ARIMA (0,2,1) model with logarithmic transformed observations without explanatory variables is used for forecasting. It may be possible to develop better forecasting models for the telex traffic between the various countries. However the main point in this example only is to illustrate the use of the weighted least squares technique.

Forecasts for 1984 based on observations from 1973 to 1983 are given in Table B-1/E.506.

H.T. [T2.506]

TABLE B-1/E.506

Forecasts for telex traffic between Germany (D), Denmark (DNK), USA (USA), Finland (FIN), Norway (NOR) and Sweden (S) in 1984

From To	D	DNK	USA	FIN	NOR	S	Sum	Forecasted sum
D	—	4869	12 | 30	2879	2397	5230	28 | 05	27 | 88
DNK	5196	—	1655	751	1270	1959	10 | 31	10 | 05
USA	11 | 03	1313	—	719	1657	2401	17 | 93	17 | 09
FIN	2655	715	741	—	489	1896	6496	6458
NOR	2415	1255	1821	541	—	1548	7580	7597
S	4828	1821	2283	1798	1333	—	12 | 63	12 | 53
Sum	26 | 97	9973	19 | 30	6688	7146	13 | 34		
Forecasted sum	26 | 97	9967	19 | 53	6659	7110	12 | 14		

Tableau B-1/E.506 [T2.506], p.

It should be noticed that there is no consistency between row and column sum forecasts and forecasts of the elements in the traffic matrix. For instance, the sum of forecasted outgoing telex traffic from Germany is 28 | 05, while the forecasted row sum is 27 | 88.

To adjust the forecasts to get consistency and to utilize both row/column forecasts and forecasts of the traffic elements the weighted least squares method is used.

B.3 *Adjustment of the traffic matrix forecasts*

To be able to use the weighted least squares method, the weights and the separate forecasts are needed as input. The separate forecasts are found in Table B-2/E.506, while the weights are based on the mean squared one step ahead forecasting errors.

Let y_t | be the traffic at time t . The ARIMA (0,2,1) model with logarithmic transformed data is given by:

$$z_t = (1 — B)^2 \mid n\, y_t = (1 — \theta B)\, a_t$$

or

$$z_t = a_t — \theta a_{t\backslash d\backslash u(em_1}$$

where

$z_t = \ln y_t — 2 \ln y_{t\backslash d\backslash u(em_1} + \ln y_{t\backslash d\backslash u(em_2}$

a_t is white noise,

θ is a parameter,

B is the backwards shift operator.

The mean squared one step ahead forecasting error of z_t | is:

MSQ =
[Formula Deleted]

where

/*ˆ$z_{t\backslash d\backslash u(em_1}$(1) is the one step ahead forecast.

The results of using the weighted least squares method is found in Table B-3/E.506 and show that the factors in Table B-1/E.506 have been adjusted. In this example only minor changes have been performed because of the high conformity in the forecasts of row/column sums and traffic elements.

H.T. [T3.506]
TABLE B-2/E.506
Inverse weights as mean as squared one step ahead forecasting

errors
of telex traffic (100↑—↑4) between
Germany
(D), Denmark
(DNK),
USA
(USA), Finland
(FIN),
Norway
(NOR) and Sweden
(S) in 1984

From To	D	DNK	USA	FIN	NOR	S	Sum
D	—	28.72	13.18	11.40	8.29	44.61	7.77
DNK	5.91	—	43.14	18.28	39.99	18.40	10.61
USA	23.76	39.19	—	42.07	50.72	51.55	21.27
FIN	23.05	12.15	99.08	—	34.41	19.96	17.46
NOR	21.47	40.16	132.57	24.64	—	17.15	20.56
S	6.38	12.95	28.60	28.08	8.76	—	6.48
Sum	6.15	3.85	14.27	9.55	12.94	8.53	

Tableau B-2/E.506 [T3.506], p.4

H.T. [T4.506]
TABLE B-3/E.506
Adjusted telex forecasts using the weighted least squares method

From To	D	DNK	USA	FIN	NOR	S	Sum
D	—	4850	12 | 84	2858	2383	5090	27 | 65
DNK	5185	—	1674	750	1257	1959	10 | 25
USA	11 | 01	1321	—	717	1644	2407	17 | 90
FIN	2633	715	745	—	487	1891	6471
NOR	2402	1258	1870	540	—	1547	7617
S	4823	1817	2307	1788	1331	—	12 | 66
Sum	26 | 44	9961	19 | 80	6653	7102	12 | 94	

Tableau B-3/E.506 [T4.506], p.5

ANNEX C
(to Recommendation E.506)

Description of a top down procedure

Let

X_T be the traffic forecast on an aggregated level,

X_i be the traffic forecast to country i ,

s_T the estimated standard deviation of the aggregated forecast,

s_i the estimated standard deviation of the forecast to country i .

Usually

$$X_T \neq \sum_i X_i, \tag{C-1}$$

so that it is necessary to find a correction

$[X'_i]$ of $[X_i]$ and $[X'_T]$ of $[X_T]$

by minimizing the expression

subject to

where α and $[\alpha_i]$ are chosen to be

The solution of the optimization problem gives the values $[X'_i]$:

A closer inspection of the data base may result in other expressions for the coefficients $[\alpha_i]$, $i = 0, 1, \ldots$ On some occasions, it will also be reasonable to use other criteria for finding the corrected forecasting values $[X'_i]$. This is shown in the top down example in Annex D.

If, on the other hand, the variance of the top forecast X_T is fairly small, the following procedure may be chosen:

The corrections $[X_i]$ are found by minimizing the expression

subject to

If α_i, $i = 1, 2, \ldots$ is chosen to be the inverse of the estimated variances, the solution of the optimization problem is given by

ANNEX D

(to Recommendation E.506)

Example of a top down modelling method

The model for forecasting telephone traffic from Norway to the European countries is divided into two separate parts. The first step is an econometric model for the total traffic from Norway to Europe. Thereafter, we apply a model for the breakdown of the total traffic on each country.

D.1 *Econometric model of the total traffic from Norway to Europe*

With an econometric model we try to explain the development in telephone traffic, measured in charged minutes, as a function of the main explanatory variables. Because of the lack of data for some variables, such as tourism, these variables have had to be omitted in the model.

The general model may be written:

$$X_t = e^K \times GNP_t^a \times P_t^b \times A_t^c \times e^{u_t} \qquad (t = 1, 2, \ldots, N) \tag{D-1}$$

where:

X_t is the demand for telephone traffic from Norway to Europe at time t (charged minutes).

GNP_t is the gross national product in Norway at time t (real prices).

P_t is the index of charges for traffic from Norway to Europe at time t (real prices).

A_t is the percentage direct-dialled telephone traffic from Norway to Europe (to take account of the effect of automation). For statistical reasons (i.e. impossibility of taking logarithm of zero) A_t goes from 1 to 2 instead of from 0 to 1.

K is the constant.

a is the elasticity with respect to *GNP* .

b is the price elasticity.

c is the elasticity with respect to automation.

u_t is the stochastic variable, summarizing the impact of those variables that are not explicitly introduced in the model and whose effects tend to compensate each other (expectation of $u_t = 0$ and var $u_t = \sigma^2$).

By applying regression analysis (OLSQ) we have arrived at the coefficients (elasticities) in the forecasting model for telephone traffic from Norway to Europe given in Table D-1/E.506 (in our calculations we have used data for the period 1951-1980).

The *t* | tatistics should be compared with the Student's Distribution with *N* | (em | fId degrees of freedom, where *N* is the number of observations and *d* is the number of estimated parameters. In this example, $N = 30$ and $d = 4$.

The model ''explains'' 99.7% of the variation in the demand for telephone traffic from Norway to Europe in the period 1951-1980.

From this logarithmic model it can be seen that:

— an increase in GNP of 1% causes an increase in the telephone traffic of 2.80%,

— an increase of 1% in the charges, measured in real prices, causes a decrease in the telephone traffic of 0.26%, and

— an increase of 1% in A_t causes an increase in the traffic of 0.29%.

We now use the expected future development in charges to Europe, in GNP, and in the future automation of traffic to Europe to forecast the development in telephone traffic from Norway to Europe from the equation:

$$X_t = e^{-16.095} \times GNP_t^{2.80} \times P_t^{-0.26} \times A_t^{0.29} \times e^{u_t} \qquad \text{(D-2)}$$

H.T. [T5.506]

TABLE D-1/E.506

Coefficients	Estimated values	*t* | statistics
K	—16.095	—4.2
a	— 2.799	— 8.2
b	— 0.264	—1.0
c	— 0.290	— 2.1

Table D-1/E.506 [T5.506], p.

D.2 *Model for breakdown of the total traffic from Norway to Europe*

The method of breakdown is first to apply the trend to forecast the traffic to each country. However, we let the trend become less important the further into the period of forecast we are, i.e. we let the trend for each country converge to the increase in the total traffic to Europe. Secondly, the traffic to each country is adjusted up or down, by a percentage that is equal to all countries, so that the sum of the traffic to each country equals the forecasted total traffic to Europe from equation (D-2).

Mathematically, the breakdown model can be expressed as follows:

Calculation of the trend for country i:

$$R_{it} = b_i + a_i \times t, \quad i = 1, \ldots, 34 \quad t = 1, \ldots, N \qquad \text{(D-3)}$$

where

$R_{it} = \frac{X_{it}}{X_t}$, i.e country i 's share of the total traffic to Europe.

X_{it} is the traffic to country i at time t

X_t is the traffic to Europe at time t

t is the trend variable

a_i and b_i are two coefficients specific to country i ; i.e. a_i is country i 's trend. The coefficients are estimated by using regression analysis, and we have based calculations on observed traffic for the period 1966-1980.

The forecasted shares for country i is then calculated by

$$\hat{R}_{it} = R_{iN} + a_i \times (t - N) \times e^{-\frac{(t-5)}{0}} \qquad \text{(D-4)}$$

where N is the last year of observation, and e is the exponential function.

The factor $e^{-\frac{(t-5)}{0}}$ is a correcting factor which ensures that the growth in the telephone traffic –v'5p' to each country will converge towards the growth of total traffic to Europe after the adjustment made in Equation (D-6).

To have the sum of the countries' shares equal one, it is necessary that

$$\sum_i R_{it} = 1 \qquad \text{(D-5)}$$

This we obtain by setting the adjusted share, $\tilde{R}_{it}$, equal to

Each country's forecast traffic is then calculated by multiplying the total traffic to Europe, X_t, by each country's share of the total traffic:

$$X_{it} = \tilde{R}_{it} \times X_t \qquad \text{(D-7)}$$

D.3 *Econometric model for telephone traffic from Norway to Central and South America, Africa, Asia, and Oceania* .

For telephone traffic from Norway to these continents we have used the same explanatory variables and estimated coefficients. Instead of gross national product, our analysis has shown that for the traffic to these continents the number of telephone stations within each continent are a better and more significant explanatory variable.

After using cross-section/time-series simultaneous estimation we have arrived at the coefficients in Table D-2/E.506 for the forecasting model for telephone traffic from Norway to these continents (for each continent we have based our calculations on data for the period 1961-1980):

H.T. [T6.506]

TABLE D-2/E.506

Coefficients	Estimated values	t statistics
Charges	—1.930	—5.5
Telephone stations	— 2.009	— 4.2
Automation	— 0.5	— —

Table D-2/E.506 [T6.506], p.

We then have $R^2 = 0.96$. The model may be written:

$$X_t^k = e^{K^k} \times (TS_t^k)^{2.009} \times (P_t^k)^{1.930} \times (A_t^k)^{0.5} \qquad \text{(D-8)}$$

where

X_t^k is the telephone traffic to continent k (k = Central America, . | | , Oceania) at time t,

e^K is the constant specific to each continent. For telephone traffic from Norway to:

Central America: $K^1 =$ —11.025

South America: $K^2 =$ —12.62

Africa: $K^3 =$ —11.395

Asia: $K^4 =$ —15.02

Oceania: $K^5 =$ —13.194

TS_t^k is the number of telephone stations within continent k at time t,

P_t^k is the index of charges, measured in real prices, to continent k at time t, and

A_t^k is the percentage direct-dialled telephone traffic to continent k.

Equation (D-8) is now used — together with the expected future development in charges to each continent, future development in telephone stations on each continent and future development in automation of telephone traffic from Norway to the continent — to forecast the future development in telephone traffic from Norway to the continent.

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Recommendation E.507

MODELS FOR FORECASTING INTERNATIONAL TRAFFIC

1 Introduction

Econometric and time series model development and forecasting requires familiarity with methods and techniques to deal with a range of different situations. Thus, the purpose of this Recommendation is to present some of the basic ideas and leave the explanation of the details to the publications cited in the reference list. As such, this Recommendation is not intended to be a complete guide to econometric and time series modelling and forecasting.

The Recommendation also gives guidelines for building various forecasting models: identification of the model, inclusion of explanatory variables, adjustment for irregularities, estimation of parameters, diagnostic checks, etc.

In addition the Recommendation describes various methods for evaluation of forecasting models and choice of model.

2 Building the forecasting model

This procedure can conveniently be described as four consecutive steps. The first step consists in finding a useful class of models to describe the actual situation. Examples of such classes are simple models, smoothing models, autoregressive models, autoregressive integrated moving average (ARIMA) models or econometric models. Before choosing the class of models, the influence of external variables should be analyzed. If special external variables have significant impact on the traffic demand, one ought to include them in the forecasting models, provided enough historical data are available.

The next step is to identify one tentative model in the class of models which have been chosen. If the class is too extensive to be conveniently fitted directly to data, rough methods for identifying subclasses can be used. Such methods of model identification employ data and knowledge of the system to suggest an appropriate parsimonious subclass of models. The identification procedure may also, in some occasions, be used to yield rough preliminary estimates of the parameters in the model. Then the tentative model is fitted to data by estimating the parameters. Usually, maximum likelihood estimators or least square estimators are used.

The old Recommendation E.506 which appeared in the *Red Book* was split into two Recommendations, revised E.506 and new E.507, and considerable new material was added to both.

The next step is to check the model. This procedure is often called diagnostic checking. The object is to find out how well the model fits the data and, in case the discrepancy is judged to be too severe, to indicate possible remedies. The outcome of this step may thus be acceptance of the model if the fit is acceptable. If on the other hand it is inadequate, it is an indication that new tentative models may in turn be estimated and subjected to diagnostic checking.

In Figure 1/E.507 the steps in the model building procedure are illustrated.

Figure 1/E.507, p.

3 Various forecasting models

The objective of § 3 is to give a brief overview of the most important forecasting models. In the GAS 10 Manual on planning data and forecasting methods [5], a more detailed description of the models is given.

3.1 *Curve fitting models*

In curve fitting models the traffic trend is extrapolated by calculating the values of the parameters of some function that is expected to characterize the growth of international traffic over time. The numerical calculations of some curve fitting models can be performed by using the least squares method.

The following are examples of common curve fitting models used for forecasting international traffic:

Linear: $Y_t = a + bt$ (3-1)

Parabolic: $Y_t = a + bt + ct^2$ (3-2)

Exponential: $Y_t = ae^{t}$ (3-3)

Logistic: $Y_t =$ *[Formula Deleted]*

Gompertz: $Y_t = M(a)^{t}$ (3-5)

where

Y_t is the traffic at time t ,

a, b, c | are parameters,

M | is a parameter describing the saturation level.

The various trend curves are shown in Figures 2/E.507 and 3/E.507.

The logistic and Gompertz curves differ from the linear, parabolic and exponential curves by having saturation or ceiling level. For further study see [10].

FIGURE 2/E.507, p.9

FIGURE 3/E.507 DIMINUER LA FIGURE, p.10

3.2 *Smoothing models*

By using a smooth process in curve fitting, it is possible to calculate the parameters of the models to fit current data very well but not necessarily the data obtained from the distant past.

The best known smoothing process is that of the moving average. The degree of smoothing is controlled by the number of most recent observations included in the average. All observations included in the average have the same weight.

In addition to moving average models, there exists another group of smoothing models based on weighting the observations. The most common models are:

— simple exponential smoothing,

— double exponential smoothing,

— discounted regression,

— Holt's method, and

— Holt-Winters' seasonal models.

For example, in the method of exponential smoothing the weight given to previous observations decreases geometrically with age according to the following equation:

$$\mu_t = (1 - a)Y_t + a\,\mu_{t-1} \qquad (3\text{-}6)$$

where:

Y_t is the measured traffic at time t,

μ_t is the estimated level at time t, and

a is the discount factor [and $(1 - a)$ is the smoothing parameter].

The impact of past observations on the forecasts is controlled by the magnitude of the discount factor.

Use of smoothing models is especially appropriate for short-term forecasts. For further studies see [1], [5] and [9].

3.3 *Autoregressive models*

If the traffic demand, X_t, at time t can be expressed as a linear combination of earlier equidistant observations of the past traffic demand, the process is an autoregressive process. Then the model is defined by the expression:

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \dots + \phi_p X_{t-p} + a_t \qquad (3\text{-}7)$$

where

a_t is white noise at time t ;

?71$_k$, k = 1, . | | p are the autoregressive parameters.

The model is denoted by AR (p) since the order of the model is p .

By use of regression analysis the estimates of the parameters can be found. Because of common trends the exogenous variables (X_{t-1} , X_{t-2} , . | | X_{t-p}) are usually strongly correlated. Hence the parameter estimates will be correlated. Furthermore, significance tests of the estimates are somewhat difficult to perform.

Another possibility is to compute the empirical autocorrelation coefficients and then use the Yule-Walker equations to estimate the parameters [?71$_k$]. This procedure can be performed when the time series [X_t] are stationary. If, on the other hand, the time series are non stationary, the series can often be transformed to stationarity e.g., by differencing the series. The estimation procedure is given in Annex A, § A.1.

3.4 *Autoregressive integrated moving average (ARIMA) models*

An extention of the class of autoregressive models which include the moving average models is called autoregressive moving average models (ARMA models). A moving average model of order q is given by:

$$X_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \dots - \theta_q a_{t-q} \qquad (3\text{-}8)$$

where

a_t is white noise at time t ;

[θ_k] are the moving average parameters

Assuming that the white noise term in the autoregressive models in § 3.3 is described by a moving average model, one obtains the so-called ARMA (p , q) model:

$$X_t = \varphi_1 X_{t-1} + \varphi_2 X_{t-2} + \dots + \varphi_p X_{t-p} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \dots - \theta_q a_{t-q} \qquad (3\text{-}9)$$

The ARMA model describes a stationary time series. If the time series is non-stationary, it is necessary to difference the series. This is done as follow:

Let Y_t be the time series and B the backwards shift operator, then

$$X_t = (1 - B)_t \qquad (3\text{-}10)$$

where

d is the number of differences to have stationarity.

The new model ARIMA (p, d, q) is found by inserting equation (3-10) into equation (3-9).

The method for analyzing such time series was developed by G. | . | . Box and G. | . Jenkins [3]. To analyze and forecast such time series it is usually necessary to use a time series program package.

As indicated in Figure 1/E.507 a tentative model is identified. This is carried out by determination of necessary transformations and number of autoregressive and moving average parameters. The identification is based on the structure of the autocorrelations and partial autocorrelations.

The next step as indicated in Figure 1/E.507 is the estimation procedure. The maximum likelihood estimates are used. Unfortunately, it is difficult to find these estimates because of the necessity to solve a nonlinear system of equations. For practical purposes, a computer program is necessary for these calculations. The forecasting model is based on equation (3-9) and the process of making forecasts l time units ahead is shown in § A.2.

The forecasting models described so far are univariate forecasting models. It is also possible to introduce explanatory variables. In this case the system will be described by a transfer function model. The methods for analyzing the time series in a transfer function model are rather similar to the methods described above.

Detailed descriptions of ARIMA models are given in [1], [2], [3], [5], [11], [15] and [17].

3.5 *State space models with Kalman Filtering*

State space models are a way to represent discrete-time process by means of difference equations. The state space modelling approach allows the conversion of any general linear model into a form suitable for recursive estimation and forecasting. A more detailed description of ARIMA state space models can be found in [1].

For a stochastic process such a representation may be of the following form:

$$X_{t\d+\d1} = ?71 X_t + Z_t + \omega_t \qquad \text{(3-11)}$$

and

$$Y_t = H X_t + \nu_t \qquad \text{(3-12)}$$

where

X_t is an s-vector of state variables in period t ,

Z_t is an s-vector of deterministic events,

?71 is an $s \times s$ | ransition matrix that may, in general, depend on t ,

ω_t is an s-vector of random modelling errors,

Y_t is a d-vector of measurements in period t ,

H | s a $d \times s$ | atrix called the observation matrix, and

ν_t is a d-vector of measurement errors.

Both ω_t in equation (3-11) and ν_t in equation (3-12) are additive random sequences with known statistics. The expected value of each sequence is the zero vector and ω_t and ν_t satisfy the conditions:

$$E \left[\omega_{fIt} (\text{*w } \$\$Ei : T : j\_ \right] = Q_t \delta_{t_j} \quad \text{for all } t, j, \qquad \text{(3-13)}$$

$$E \left[\nu_{fIt} (\text{*n } \$\$Ei : T : j\_ \right] = R_t \delta_{t_j} \quad \text{for all } t, j,$$

where

Q_t and R_t are nonnegative definite matrices,

and

$\delta_{t\d j}$ is the Kronecker delta.

Q_t is the covariance matrix of the modelling errors and R_t is the covariance matrix of the measurement errors; the ω_t and the ν_t are assumed to be uncorrelated and are referred to as white noise. In other words:

$$E \left[\nu_{fIt} (\text{*w } \$\$Ei : T : j\_ \right] = 0 \quad \text{for all } t, j, \qquad \text{(3-14)}$$

and

$$E \left[\nu_{fIt} \, \text{f IX } \$\$\text{Ei} : T : 0\_ \right] = 0 \quad \text{for all } t. \qquad \text{(3-15)}$$

Under the assumptions formulated above, determine $X_{t\d,\d t}$ such that:

$$E \left[(X_{\text{f It}, t} (\text{em } X_{\text{f It}}))^{fIT} (X_{\text{f It}, t} (\text{em } X_{\text{f It}})) \right] = \text{minimum}, \qquad \text{(3-16)}$$

A matrix A is nonnegative definite, if and only if, for all vectors z, z | (>=" | .

where

$X_{t\backslash \mathrm{d},\backslash \mathrm{d}t}$ is an estimate of the state vector at time t , and

X_t is the vector of true state variables.

The Kalman Filtering technique allows the estimation of state variables recursively for on-line applications. This is done in the following manner. Assuming that there is no explanatory variable Z_t, once a new data point becomes available it is used to update the model:

$$X_{t|t} = X_{t,t-1} + K_t(Y_t - HX_{t,t-1}) \qquad (3\text{-}17)$$

where

K_t is the Kalman Gain matrix that can be computed recursively [18].

Intuitively, the gain matrix determines how much relative weight will be given to the last observed forecast error to correct it. To create a k-step ahead projection the following formula is used:

$$X_{t+k,t} = \Phi^k X_{t|t} \qquad (3\text{-}18)$$

where

$X_{t+k,t}$ is an estimate of X_{t+k} given observations $Y_1, Y_2, \ldots, Y_t$.

Equations (3-17) and (3-18) show that the Kalman Filtering technique leads to a convenient forecasting procedure that is recursive in nature and provides an unbiased, minimum variance estimate of the discrete time process of interest.

For further studies see [4], [5], [16], [18], [19] and [22].

The Kalman Filtering works well when the data under examination are seasonal. The seasonal traffic load data can be represented by a periodic time series. In this way, a seasonal Kalman Filter can be obtained by superimposing a linear growth model with a seasonal model. For further discussion of seasonal Kalman Filter techniques see [6] and [20].

3.6 *Regression models*

The equations (3-1) and (3-2) are typical regression models. In the equations the traffic, Y_t, is the dependent (or explanatory) variable, while time t is the independent variable.

A regression model describes a linear relation between the dependent and the independent variables. Given certain assumptions ordinary least squares (OLS) can be used to estimate the parameters.

A model with several independent variables is called a multiple regression model. The model is given by:

$$Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \ldots + \beta_k X_{kt} + u_t \qquad (3\text{-}19)$$

where

Y_t is the traffic at time t,

β_i, $i = 0, 1, \ldots, k$ are the parameters,

X_{it}, $i = 1, 2, \ldots, k$ is the value of the independent variables at time t,

u_t is the error term at time t.

Independent or explanatory variables which can be used in the regression model are, for instance, tariffs, exports, imports, degree of automation. Other explanatory variables are given in § 2 ‘‘Base data for forecasting’’ in Recommendation E.506.

Detailed descriptions of regression models are given in [1], [5], [7], [15] and [23].

3.7 *Econometric models*

Econometric models involve equations which relate a variable which we wish to forecast (the dependent or endogenous variable) to a number of socio-economic variables (called independent or explanatory variables). The form of the equations should reflect an expected casual relationship between the variables. Given an assumed model form, historical or cross sectional data are used to estimate coefficients in the equation. Assuming the model remains valid over time, estimates of future values of the independent variables can be used to give forecasts of the variables of interest. An example of a typical econometric model is given in Annex C.

There is a wide spectrum of possible models and a number of methods of estimating the coefficients (e.g., least squares, varying parameter methods, nonlinear regression, etc.). In many respects the family of econometric models available is far more flexible than other models. For example, lagged effects can be incorporated, observations weighted, ARIMA residual models subsumed, information from separate sections pooled and parameters allowed to vary in econometric models, to mention a few.

One of the major benefits of building an econometric model to be used in forecasting is that the structure or the process that generates the data must be properly identified and appropriate causal paths must be determined. Explicit structure identification makes the source of errors in the forecast easier to identify in econometric models than in other types of models.

Changes in structures can be detected through the use of econometric models and outliers in the historical data are easily eliminated or their influence properly weighted. Also, changes in the factors affecting the variables in question can easily be incorporated in the forecast generated from an econometric model.

Often, fairly reliable econometric models may be constructed with less observations than that required for time series models. In the case of pooled regression models, just a few observations for several cross-sections are sufficient to support a model used for predictions.

However, care must be taken in estimating the model to satisfy the underlying assumptions of the techniques which are described in many of the reference works listed at the end of this Recommendation. For example the number of independent variables which can be used is limited by the amount of data available to estimate the model. Also, independent variables which are correlated to one another should be avoided. Sometimes correlation between the variables can be avoided by using differenced or detrended data or by transformation of the variables. For further studies see [8], [12], [13], [14] and [21].

4 Discontinuities in traffic growth

4.1 *Examples of discontinuities*

It may be difficult to assess in advance the magnitude of a discontinuity. Often the influence of the factors which cause discontinuities is spread over a transitional period, and the discontinuity is not so obvious. Furthermore, discontinuities arising, for example, from the introduction of international subscriber dialling are difficult to identify accurately, because changes in the method of working are usually associated with other changes (e.g. tariff reductions).

An illustration of the bearing of discontinuities on traffic growth can be observed in the graph of Figure 4/E.507.

Discontinuities representing the doubling — and even more — of traffic flow are known. It may also be noted that changes could occur in the growth trend after discontinuities.

In short-term forecasts it may be desirable to use the trend of the traffic between discontinuities, but for long-term forecasts it may be desirable to use a trend estimate which is based on long-term observations, including previous discontinuities.

In addition to random fluctuations due to unpredictable traffic surges, faults, etc., traffic measurements are also subject to systematic fluctuations, due to daily or weekly traffic flow cycles, influence of time differences, etc.

4.2 *Introduction of explanatory variables*

Identification of explanatory variables for an econometric model is probably the most difficult aspect of econometric model building. The explanatory variables used in an econometric model identify the main sources of influence on the variable one is concerned with. A list of explanatory variables is given in Recommendation E.506, § 2.

Figure 4/E.507, p.

Economic theory is the starting point for variable selection. More specifically, demand theory provides the basic framework for building the general model. However, the description of the structure or the process generating the data often dictate what variables enter the set of explanatory variables. For instance, technological relationships may need to be incorporated in the model in order to appropriately define the structure.

Although there are some criteria used in selecting explanatory variables [e.g., $\bar{R}^2$, Durbin-Watson (D-W) statistic, root mean square error (RMSE), ex-post forecast performance, explained in the references], statistical problems and/or availability of data (either historical or forecasted) limit the set of potential explanatory variables and one often has to revert to proxy variables. Unlike pure statistical models, econometric models admit explanatory variables, not on the basis of statistical criteria alone but, also, on the premise that causality is, indeed, present.

A completely specified econometric model will capture turning points. Discontinuities in the dependent variable will not be present unless the parameters of the model change drastically in a very short time period. Discontinuities in the growth of telephone traffic are indications that the underlying market or technological structure have undergone large changes.

Sustained changes in the growth of telephone demand can either be captured through varying parameter regression or through the introduction of a variable that appears to explain the discontinuity (e.g., the introduction of an advertising variable if advertising is judged to be the cause of the structural change). Once-and-for-all, or step-wise discontinuities, cannot be handled by the introduction of explanatory changes: dummy variables can resolve this problem.

4.3 *Introduction of dummy variables*

In econometric models, qualitative variables are often relevant; to measure the impact of qualitative variables , dummy variables are used. The dummy variable technique uses the value 1 for the presence of the qualitative attribute that has an impact on the dependent variable and 0 for the absence of the given attribute.

Thus, dummy variables are appropriate to use in the case where a discontinuity in the dependent variable has taken place. A dummy variable, for example, would take the value of zero during the historical period when calls were operator handled and one for the period for which direct dial service is available.

Dummy variables are often used to capture seasonal effects in the dependent variable or when one needs to eliminate the effect of an outlier on the parameters of a model, such as a large jump in telephone demand due to a postal strike or a sharp decline due to facility outages associated with severe weather conditions.

Indiscriminate use of dummy variables should be discouraged for two reasons:

1) dummy variables tend to absorb all the explanatory power during discontinuities, and

2) they result in a reduction in the degrees of freedom.

5 Assessing model specification

5.1 *General*

In this section methods for testing the significance of the parameters and also methods for calculating confidence intervals are presented for some of the forecasting models given in § 3. In particular the methods relating to regression analysis and time series analysis will be discussed.

All econometric forecasting models presented here are described as regression models. Also the curve fitting models given in § 3.1 can be described as regression models.

An exponential model given by

$$Z_t = ae^{bt} \cdot u_t \tag{5-1}$$

may be transformed to a linear form

$$\ln Z_t = \ln a + bt + \ln u_t \tag{5-2}$$

or

$$Y_t = \beta_0 + \beta_1 X_t + a_t \tag{5-3}$$

where

Y_t = $\ln Z_t$

β_0 = $\ln a$

β_1 = b

X_t = t

a_t = $\ln u_t$ (white noise).

5.2 *Autocorrelation*

A good forecasting model should lead to small autocorrelated residuals. If the residuals are significantly correlated, the estimated parameters and also the forecasts may be poor. To check whether the errors are correlated, the autocorrelation function r_k, $k = 1, 2, . | \ |$ is calculated. r_k is the estimated autocorrelation of residuals at lag k . A way to detect autocorrelation among the residuals is to plot the autocorrelation function and to perform a Durbin-Watson test. The Durbin-Watson statistic is:

where

e_t is the estimated residual at time t ,

N is the number of observations.

5.3 *Test of significance of the parameters*

One way to evaluate the forecasting model is to analyse the impact of different exogenous variables. After estimating the parameters in the regression model, the significance of the parameters has to be tested.

In the example of an econometric model in Annex C, the estimated values of the parameters are given. Below these values the estimated standard deviation is given in parentheses. As a rule of thumb, the parameters are considered as significant if the absolute value of the estimates exceeds twice the estimated standard deviation. A more accurate way of testing the significance of the parameters is to take into account the distributions of their estimators.

The multiple correlation coefficient (or coefficient of determination) may be used as a criterion for the fitting of the equation.

The multiple correlation coefficient, R^2, is given by:

If the multiple correlation coefficient is close to 1 the fitting is satisfactory. However, a high R^2 does not imply an accurate forecast.

In time series analysis, the discussion of the model is carried out in another way. As pointed out in § 3.4, the number of autoregressive and moving average parameters in an ARIMA model is determined by an identification procedure based on the structure of the autocorrelation and partial autocorrelation function.

The estimation of the parameters and their standard deviations is performed by an iterative nonlinear estimation procedure. Hence, by using a time series analysis computer program, the estimates of the parameters can be evaluated by studying the estimated standard deviations in the same way as in regression analysis.

An overall test of the fitting is based on the statistic

where r_i is the estimated autocorrelation at lag i and d is the number of parameters in the model. When the model is adequate, Q_{N-d} is approximately chi-square distributed with $N - d$ degrees of freedom. To test the fitting, the value Q_{N-d} can be compared with fractiles of the chi-square distribution.

5.4 *Validity of exogenous variables*

Econometric forecasting models are based on a set of exogenous variables which explain the development of the endogenous variable (the traffic demand). To make forecasts of the traffic demand, it is necessary to make forecasts of each of the exogenous variables. It is very important to point out that an exogenous variable should not be included in the forecasting model if the prediction of the variable is less confident than the prediction of the traffic demand.

Suppose that the exact development of the exogenous variable is known which, for example, is the case for the simple models where time is the explanatory variables. If the model fitting is good and the white noise is normally distributed with expectation equal to zero, it is possible to calculate confidence limits for the forecasts. This is easily done by a computer program.

On the other hand, the values of most of the explanatory variables cannot be predicted exactly. The confidence of the prediction will then decrease with the number of periods. Hence, the explanatory variables will cause the confidence interval of the forecasts to increase with the number of the forecast periods. In these situations it is difficult to calculate a confidence interval around the forecasted values.

If the traffic demand can be described by an autoregressive moving average model, no explanatory variables are included in the model. Hence, if there are no explanatory variable in the model, the confidence limits of the forecasting values can be calculated. This is done by a time series analysis program package.

5.5 *Confidence intervals*

Confidence intervals, in the context of forecasts, refer to statistical constructs of forecast bounds or limits of prediction. Because statistical models have errors associated with them, parameter estimates have some variability associated with their values. In other words, even if one has identified the correct forecasting model, the influence of endogenous factors will cause errors in the parameter estimates and the forecast. Confidence intervals take into account the uncertainty associated with the parameter estimates.

In causal models, another source of uncertainty in the forecast of the series under study are the predictions of the explanatory variables. This type of uncertainty cannot be handled by confidence intervals and is usually ignored, even though it may be more significant than the uncertainty associated with coefficient estimates. Also, uncertainty due to possible outside shocks is not reflected in the confidence intervals.

For a linear, static regression model, the confidence interval of the forecast depends on the reliability of the regression coefficients, the size of the residual variance, and the values of the explanatory variables. The 95% confidence interval for a forecasted value Y_{N+1} is given by:

$$\hat{Y}_N(1) - 2\hat{s} \quad Y_{N+1} \quad \hat{Y}_N(1) + 2\hat{s} \tag{5-7}$$

where $\hat{Y}_N(1)$ is the forecast one step ahead and $\hat{s}$ is the standard error of the forecast.

This says that we expect, with a 95% probability, that the actual value of the series at time $N + 1$ will fall within the limits given by the confidence interval, assuming that there are no errors associated with the forecast of the explanatory variables.

6 Comparison of alternative forecasting models

6.1 *Diagnostic check — Model evaluation*

Tests and diagnostic checks are important elements in the model building procedure. The quality of the model is characterized by the residuals. Good forecasting models should lead to small autocorrelated residuals, the variance of the residuals should not decrease or increase and the expectation of the residuals should be zero or close to zero. The precision of the forecast is affected by the size of the residuals which should be small.

In addition the confidence limits of the parameter estimates and the forecasts should be relatively small. And in the same way, the mean square error should be small compared with results from other models.

6.2 *Forecasts of levels versus forecasts of changes*

Many econometric models are estimated using levels of the dependent and independent variables. Since economic variables move together over time, high coefficients of determination are obtained. The collinearity among the levels of the explanatory variables does not present a problem when a model is used for forecasting purposes alone, given that the collinearity pattern in the past continues to exist in the future. However, when one attempts to measure structural coefficients (e.g., price and income elasticities) the collinearity of the explanatory variables (known as multicollinearity) renders the results of the estimated coefficients unreliable.

To avoid the multicollinearity problem and generate benchmark coefficient estimates and forecasts, one may use changes of the variables (first difference or first log difference which is equivalent to a percent change) to estimate a model and forecast from that model. Using changes of variables to estimate a model tends to remove the effect of multicollinearity and produce more reliable coefficient estimates by removing the common effect of economic influences on the explanatory variables.

By generating forecasts through levels of and changes in the explanatory variables, one may be able to produce a better forecast through a reconciliation process. That is, the models are adjusted so that the two sets of forecasts give equivalent results.

6.3 *Ex-post forecasting*

Ex-post forecasting is the generation of a forecast from a model estimated over a sub-sample of the data beginning with the first observation and ending several periods prior to the last observation. In ex-post forecasting, actual values of the explanatory variables are used to generate the forecast. Also, if forecasted values of the explanatory variables are used to produce an ex-post forecast, one can then measure the error associated with incorrectly forecasted explanatory variables.

The purpose of ex-post forecasting is to evaluate the forecasting performance of the model by comparing the forecasted values with the actuals of the period after the end of the sub-sample to the last observation. With ex-post forecasting, one is able to assess forecast accuracy in terms of:

1) percent deviations of forecasted values from actual values,

2) turning point performance,

3) systematic behaviour of deviations.

Deviations of forecasted values from actual values give a general idea of the accuracy of the model. Systematic drifts in deviations may provide information for either re-specifying the model or adjusting the forecast to account for the drift in deviations. Of equal importance in evaluating forecast accuracy is turning point performance, that is, how well the model is able to forecast changes in the movement of the dependent variable. More criteria for evaluating forecast accuracy are discussed below.

6.4 *Forecast performance criteria*

A model might fit the historical data very well. However, when the forecasts are compared with future data that are not used for estimation of parameters, the fit might not be so good. Hence comparison of forecasts with actual observations may give additional information about the quality of the model. Suppose we have the time series, $Y_1, Y_2, \ldots, Y_N, Y_{N+1}, \ldots, Y_{N+M}$.

The M last observations are removed from the time series and the model building procedure. The one-step-ahead forecasting error is given by:

$$e_{N+t} = Y_{N+t} - \hat{Y}_{N+t-1}(1) \qquad t = 1, 2, \ldots, M \tag{6-1}$$

where

$\hat{Y}_{N+t-1}(1)$ is the one-step-ahead forecast.

Mean error

The mean error, ME, is defined by

ME is a criterium for forecast bias. Since the expectation of the residuals should be zero, a large deviation from zero indicates bias in the forecasts.

Mean percent error

The mean percent error, MPE, is defined by

This statistic also indicates possible bias in the forecasts. The criterium measures percentage deviation in the bias. It is not recommended to use MPE when the observations are small.

Root mean square error

The root mean square error, RMSE, of the forecast is defined as

RMSE is the most commonly used measure for forecasting precision.

Mean absolute error

The mean absolute error, MAE, is given by

Theil's inequality coefficient

Theil's inequality coefficient is defined as follows:

Theil's *U* is preferred as a measure of forecast accuracy because the error between forecasted and actual values can be broken down to errors due to:

1) central tendency,
2) unequal variation between predicted and realized changes, and
3) incomplete covariation of predicted and actual changes.

This decomposition of prediction errors can be used to adjust the model so that the accuracy of the model can be improved.

Another quality that a forecasting model must possess is ability to capture turning points. That is, a forecast must be able to change direction in the same time period that the actual series under study changes direction. If a model is estimated over a long period of time which contains several turning points, ex-post forecast analysis can generally detect a model's inability to trace closely actuals that display turning points.

7 Choice of forecasting model

7.1 *Forecasting performance*

Although the choice of a forecasting model is usually guided by its forecasting performance, other considerations must receive attention. Thus, the length of the forecast period, the functional form, and the forecast accuracy of the explanatory variables of an econometric model must be considered.

The length of the forecast period affects the decision to use one type of a model versus another, along with historical data limitations and the purpose of the forecasting model. For instance, ARIMA models may be appropriate forecasting models for short-term forecasts when stability is not an issue, when sufficient historical data are available, and when causality is not of interest. Also, when the structure that generates the data is difficult to identify, one has no choice but to use a forecasting model which is based on historical data of the variable of interest.

The functional form of the model must also be considered in a forecasting model. While it is true that a more complex model may reduce the model specification error, it is also true that it will, in general, considerably increase the effect of data errors. The model form should be chosen to recognize the trade-off between these sources of error.

Availability of forecasts for explanatory variables and their reliability record is another issue affecting the choice of a forecasting model. A superior model using explanatory variables which may not be forecasted accurately can be inferior to an average model whose explanatory variables are forecasted accurately.

When market stability is an issue, econometric models which can handle structural changes should be used to forecast. When causality matters, simple models or ARIMA models cannot be used as forecasting tools. Nor can they be used when insufficient historical data exist. Finally, when the purpose of the model is to forecast the effects associated with changes in the factors that influence the variable in question, time series models may not be appropriate (with the exception, of course, of transfer function and multiple time series models).

7.2 *Length of forecast period*

For normal extensions of switching equipment and additions of circuits, a forecast period of about six years is necessary. However, a longer forecast period may be necessary for the planning of new cables or other transmission media or for major plant installations. Estimates in the long term would necessarily be less accurate than short-term forecasts but that would be acceptable.

In forecasting with a statistical model, the length of the forecast period is entirely determined by:

a) the historical data available,

b) the purpose or use of the forecast,

c) the market structure that generates the data,

d) the forecasting model used,

e) the frequency of the data.

The historical data available depends upon the period over which it has been collected and the frequency of collection (or the length of the period over which data is aggregated). A small historical data base can only support a short prediction interval. For example, with 10 or 20 observations

a model can be used to forecast 4-5 periods past the sample (i.e. into the future). On the other hand, with 150-200 observations, potentially reliable forecasts can be obtained for 30 to 50 periods past the sample — other things being equal.

Certainly, the purpose of the forecast affects the number of predicted periods. Long range facility planning requires forecasts extending 15-20 or more years into the future. Rate change evaluations may only require forecasts for 2-3 years. Alteration of routing arrangements could only require forecasts extending a few months past the sample.

Stability of a market, or lack thereof, also affect the length of the forecast period. With a stable market structure one could conceivably extend the forecast period to equal the historical period. However, a volatile market does not afford the same luxury to the forecaster; the forecast period can only consist of a few periods into the future.

The forecasting models used to generate forecasts do, by their nature, influence the decision on how far into the future one can reasonably forecast. Structural models tend to perform better than other models in the long run, while for short-run predictions all models seem to perform equally well.

It should be noted that while the purpose of the forecast and the forecasting model affect the length of the forecast, the number of periods to be forecasted play a crucial role in the choice of the forecasting model and the use to which a forecast is put.

ANNEX A

(to Recommendation E.507)

Description of forecasting procedures

A.1 *Estimation of autoregressive parameters*

The empirical autocorrelation at lag k is given by:

$$r_k = \frac{R_k}{R_0} \tag{A-1}$$

where

and

N being the total number of observations.

The relation between $[r_k]$ and the estimates $[\hat{\gamma}_k]$ of $[\gamma_k]$ is given by the Yule-Walker equations :

$$\begin{aligned} r_1 &= \hat{\gamma}_1 + \hat{\gamma}_2 r_1 + \dots + \hat{\gamma}_p r_{p-1} \\ r_2 &= \hat{\gamma}_1 r_1 + \hat{\gamma}_2 r_2 + \dots + \hat{\gamma}_p r_{p-2} \\ &\times \\ &\times \\ r_p &= \hat{\gamma}_1 r_{p-1} + \hat{\gamma}_2 r_{p-2} + \dots + \hat{\gamma}_p \end{aligned} \tag{A-4}$$

Hence the estimators $[\hat{\gamma}_k]$ can be found by solving this system of equations.

For computations, an alternative to directly solving the equations is the following recursive procedure. Let $[\hat{\gamma}_{k,j}]_j$ be estimators of the parameters at lag $j = 1, 2, \dots, k$ given that the total number of parameters are k. The estimators $[\hat{\gamma}_{k+1,j}]_j$ are then found by

Defining $\hat{\gamma}_{p,j} = \hat{\gamma}_j$, $j = 1, 2, \dots, p$, the forecast of the traffic demand at time $t+1$ is expressed by:

$$\hat{X}_{t+1} = \hat{\gamma}_1 X_t + \hat{\gamma}_2 X_{t-1} + \dots + \hat{\gamma}_p X_{t-p} \tag{A-7}$$

A.2 *Forecasting with ARIMA models*

The forecast l time units ahead is given by:

which means that $[X_j]$ is defined as a forecast when $j > t$ and otherwise as an actual observation and that $[a_j]$ is defined as 0 when $j > t$ since white noise has expectation 0. If the observations are known (j t), then $[a_j]$ is equal to the residual.

ANNEX B

(to Recommendation E.507)

Kalman Filter for a linear trend model

To model telephone traffic, it is assumed that there are no deterministic changes in the demand pattern. This situation can be modelled by setting the deterministic component Z_t to zero. Then the general state space model is:

$$X_{t+1} = \phi X_t + \omega_t \qquad \text{(B-1)}$$
$$Y_t = HX_t + \nu_t$$

where

X_t is an s-vector of state variables in period t ,

Y_t is a vector of measurements in year t ,

ϕ is an $s \times s$ transition matrix that may, in general, depend on t ,

and

ω_t is an s-vector of random modelling errors,

ν_t is the measurement error in year t .

For modelling telephone traffic demand, adapt a simple two-state, one-data variable model defined by:

and

$$y_t = x_t + \nu_t \qquad \text{(B-3)}$$

where

x_t is the true load in year t ,

$\mathring{x}_t$ is the true incremental growth in year t ,

y_t is the measured load in year t ,

ν_t is the measurement error in year t .

Thus, in our model

The one-step-ahead projection is written as follows:

where

$X_{t+1,t}$ is the projection of the state variable in period $t + 1$ given observations through year t.

The α_t and β_t coefficients are the Kalman gain matrices in year t. Rewriting the above equation yields:

$$x_{t,t} = (1-\alpha_t)x_{t,t-1} \quad \alpha_t \tag{B-6}$$

and

$$\hat{x}_{t,t} = (1-\beta_t)\hat{x} + \beta_t(y_t - x_{t-1,t-1}) \tag{B-7}$$

The Kalman Filter creates a linear trend for each time series being forecast based on the current observation or measurement of traffic demand and the previous year's forecast of that demand. The observation and forecasted traffic load are combined to produce a smoothed load that corresponds to the level of the process, and a smoothed growth increment. The Kalman gain values α_t and β_t can be either fixed or adaptive. In [16] Moreland presents a method for selecting fixed, robust parameters that provide adequate performance independent of system noise, measurement error, and initial conditions. For further details on the proper selection of these parameters see [6], [20] and [22].

ANNEX C

(to Recommendation E.507)

Example of an econometric model

To illustrate the workings of an econometric model, we have chosen the model of United States billed minutes to Brazil. This model was selected among alternative models for three reasons:

a) to demonstrate the introduction of explanatory variables,

b) to point out difficulties associated with models used for both the estimation of the structure and forecasting purposes, and

c) to show how transformations may affect the results.

The demand of United States billed minutes to Brazil (*MIN*) is estimated by a log-linear equation which includes United States billed messages to Brazil (*MSG*), a real telephone price index (*RPI*), United States personal income in 1972 prices (*YP* 72), and real bilateral trade between the United States and Brazil (*RTR*) as explanatory variables. This model is represented as:

$$\ln(MIN)_t = \beta_0 + \beta_1 \ln(MSG)_t + \beta_2 \ln(RPI)_t + \beta_3 \ln(YP\,72)_t + \beta_4 \ln(RTR)_t + u_t \tag{C-1}$$

where u_t is the error term of the regression and where, $\beta_1 > 0$, $\beta_2 < 0$, $\beta_3 > 0$ and $\beta_4 > 0$ are expected values.

Using ridge regression to deal with severe multicollinearity problems, we estimate the equation over the 1971 | | (i.e. first quarter of 1971) to 1979 | | interval and obtain the following results:

$\ln(MIN)_t = —3.489 + (0.619) \ln(MSG)_t — (0.447) \ln(RPI)_t + (1.166) \ln(YP\ 72)_t + (0.281) \ln(RTR)_t$ $\ln(MIN)_t = —3.489 + (0.035) \ln(MSG)_t — (0.095) \ln(RPI)_t + (0.269) \ln(YP\ 72)_t + (0.084)$ (C-2)

.sp 1

$$R^2 = 0.985, SER = 0.083, D\text{-}W = 0.922, k = 0.10$$

(C-3)

where R^2 is the adjusted coefficient of determination, *SER* is the standard error of the regression, D-W is the Durbin-Watson statistic, and k is the ridge regression constant. The values in parentheses under the equation are the estimated standard deviation of the estimated parameters β_1, β¾d β_2 β_3, β_4.

The introduction of messages as an explanatory variable in this model was necessitated by the fact that since the mid-seventies transmission quality has improved and completion rates have risen while, at the same time, the strong growth in this market has begun to dissipate. Also, the growth rates for some periods could not have been explained by rate activity on either side or real United States personal income. The behaviour of the message variable in the minute equation was able to account for all these factors.

Because the model serves a dual purpose — namely, structure estimation and forecasting — at least one more variable is introduced than if the model were to be used for forecasting purposes alone. The introduction of additional explanatory variables results in severe multicollinearity and necessitates employing ridge regression which lowers R^2 and the Durbin-Watson statistic. Consequently, the predictive power of the model is reduced somewhat.

The effect of transforming the variables of a model are shown in the ex-post forecast analysis performed on the model of United States billed minutes to Brazil. The deviations using levels of the variables are larger than those of the logarithms of the variables which were used to obtain a better fit (the estimated RMSE for the log-linear regression model is 0.119 | 27). The forecast results in level and logarithmic form are shown in Table C-1/E.507.

H.T. [T1.507]

TABLE C-1/E.507

	Logarithms			Levels		
	Forecast	Actual	% deviation	Forecast	Actual	% deviation
1980: 1	14.858	14.938	—0.540	2 | 36 | 69	3 | 73 | 97	— 7.725
2	14.842	14.972	—0.872	2 | 91 | 50	3 | 80 | 34	—12.234
3	14.916	15.111	—1.296	3 | 05 | 37	3 | 54 | 92	—17.746
4	14.959	15.077	—0.778	3 | 37 | 98	3 | 29 | 16	—11.089
1981: 1	15.022	15.102	—0.535	3 | 41 | 33	3 | 21 | 35	— 7.731
2	14.971	15.141	—1.123	3 | 75 | 77	3 | 62 | 92	—15.601
3	15.395	15.261	— 0.879	4 | 52 | 78	4 | 44 | 78	14.333
4	15.405	15.302	— 0.674	4 | 01 | 46	4 | 21 | 55	— 10.844
1982: 1	15.365	15.348	— 0.110	4 | 09 | 65	4 | 30 | 38	— 1.702
2	15.326	15.386	—0.387	4 | 28 | 47	4 | 07 | 01	— 5.802

Table C-1/E.507 [T1.507] p.

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Recommendation E.508

FORECASTING NEW INTERNATIONAL SERVICES

1 Introduction

The operation and administration of an international telecommunications network should include the consideration of subscriber demands for new services which may have different characteristics than the traditional traffic (i.e. peak busy hours, bandwidth requirements, and average call durations may be different). By addressing these new demands, Administrations can be more responsive to customer requirements for innovative telecommunications services. Based on the type of service and estimated demand for a service, network facilities and capacity may have to be augmented. An augmentation of the international network could require large capital investments and additional administrative functions and responsibilities. Therefore, it is appropriate that Administrations forecast new international services within their planning process.

This Recommendation presents methods for forecasting new services. The definitions of some of the characteristics of these services, together with their requirements, are covered in § 2, followed by base data requirements in § 3. § 4 discusses research to identify the potential market. Presentation of forecasting methods are contained in § 5. § 6 concludes with forecast tests and adjustments.

2 New service definitions

2.1 A distinction exists between those services which are enhancements of existing services carried on the existing network and those services which are novel.

Many of the services in this latter category will be carried on the Integrated Services Digital Network (ISDN). It is not the purpose of this section to provide an exhaustive list of services but rather to establish a framework for their classification. This framework is required because different base data and forecasting strategies may be necessary in each case.

2.2 enhanced services offered over the existing network

These are services which are offered over the existing network, and which offer an enhancement of the original use for which the network was intended. Services such as the international freephone service, credit card calling and closed user groups are examples of enhancements of voice services; while facsimile, telefax and videotex are examples of non-voice services. These services may be carried over the existing network and, therefore, data will concern usage or offered load specific to the enhancement. Arrangements can be established for the measurement of this traffic, such as the use of special network access codes for non-voice applications or by sampling outgoing circuits for the proportion of non-voice to voice traffic.

2.3 novel services

Novel services are defined as totally new service offerings many of which may be carried over the ISDN. In the case of ISDN, Recommendation I.210 divides telecommunications services into two broad categories: bearer services and teleservices. Recommendation I.210 further defines supplementary services which modify or supplement a basic telecommunications service. The definition of bearer services supported by the

ISDN is contained in Recommendations I.210 and I.211, while that for teleservices is found in Recommendations I.210 and I.212. Bearer services may include circuit switched services from 64 kbit/s to 2 Mbit/s and packet services. Circuit switched services above 2 Mbit/s are for further study.

Teleservices may include Group 4 facsimile, mixed mode text and facsimile, 64 kbit/s Teletex and Videotex, videophone, videoconferencing, electronic funds transfer and point of sale transaction services. These lists are not exhaustive but indicate the nature and scope of bearer services and teleservices. Examples of new services are diagrammatically presented in Table 1/E.508.

H.T. [T1.508]
TABLE 1/E.508
Examples of enhanced and novel services

{	‘‘Novel’’ services Bearer services	Teleservices
Teletex Facsimile Videotex Message handling systems International freephone Credit cards Closed user groups Teletex (64 kbit/s) Videotex (64 kbit/s) }	Packet Circuit switched services — 64 kbit/s — 2 Mbit/s	Group 4 facsimile Mixed mode Videophone Videoconferencing Electronic funds transfer Point of sale transactions {

Table 1/E.508 [T1.508], p.

3 Base data for forecasting

3.1 *Measurement of enhanced services*

Measurements for existing services are available in terms of calls, minutes, Erlangs, etc. These procedures are covered in Recommendation E.506, § 2. In order to measure/identify enhanced service data

from other traffic data on the same network it may be necessary to establish sampling or other procedures to aid in the estimation of this traffic, as described in § 4 and § 5.

3.2 *Novel services*

Novel services, as defined in § 2, may be carried on the ISDN. In the case of the ISDN, circuit switched bearer services and their associated teleservices will be measured in 64 kbit/s increments. Packet switched bearer services and associated teleservices will be measured by a unit of throughput, for example, kilocharacters or kilopackets per second. Other characteristics needed will reflect service quality measurements such as: noise, echo, post-dialing delay, clipping, bit-error rate, holding time, set-up time, error-free seconds, etc.

4 Market research

Market research is conducted to test consumer response and behaviour. This research employs the methods of questionnaires, market analysis, focus groups and interviews. Its purpose is to determine consumers’ intentions to purchase a service, attitudes towards new and existing services, price sensitivity and cross service elasticities. Market research helps make

decisions concerning which new services should be developed. A combination of the qualitative and quantitative phases of market research can be used in the initial stages of forecasting the demand for a new service.

The design of market research considers a sampling frame, customer/market stratification, the selection of a statistically random sample and the correction of results for non-response bias. The sample can be drawn from the entire market or from subsegments of the market. In sampling different market segments, factors which characterize the segments must be alike with respect to consumer behaviour (small intragroup variance) and should differ as much as possible from other segments (large intergroup variance); each segment is homogeneous while different segments are heterogeneous.

The market research may be useful in forecasting existing services or the penetration of new services. The research may be used in forecasting novel services or any service which has no historical series of demand data. It is important that potential consumers be given a complete description of the new service, including the terms and conditions which would accompany its provisioning. It is also important to ask the surveyees whether they would purchase the new service under a variety of illustrative tariff structures

and levels. This aspect of market research will aid in redimensioning the demand upon final determination of the tariff structure and determining the customers' initial price sensitivity.

5 Forecasting procedures

5.1 *General*

The absence of historical data is the fundamental difference between forecasting new services and forecasting existing services. The forecast methodology is dependent on the base data. For example, for a service that is planned but has not been introduced, market research survey data can be used. If the service is already in existence in some countries, forecasting procedures for its introduction to a new country will involve historical data on other countries, its application to the new country and comparison of characteristics between countries.

5.2 *Sampling and questionnaire design*

The forecasting procedure for novel services based on market research is made up of five consecutive steps. The first of these consists in defining the scope of the study.

The second step involves the definition and selection of a sample from the population, where the population includes all potential customers which can be identified by qualitative market research developed through interviews at focus groups. The research can use stratified samples which involves grouping the population into homogeneous segments (or strata) and then sampling within each strata. Stratification prevents the disproportionate representation of some parts of the population that can result by chance with simple random sampling. The sample can be structured to include specified numbers of respondents having characteristics that are known, or believed, to affect the subject of the research. Examples of customer characteristics would be socio-economic background and type of business.

The third step is the questionnaire design. A trade-off exists between obtaining as much information as practical and limiting the questionnaire to a reasonable length, as determined by the surveyor. Most questionnaires have three basic sections:

1) qualifying questions to determine if a knowledgeable person has been contacted;

2) basic questions including all questions which constitute the body of the questionnaire;

3) classification questions collecting background on demographic information.

The fourth step involves the implementation of the research — the actual surveying portion. Professional interviewers, or firms specializing in market research should be employed for interviewing.

The fifth and final step is the tabulation and analysis of the survey data. § 5.3-5.7 describe this process in detail.

5.3 *Conversion ratios for the sample*

Conversion ratios are used in estimating the proportion of respondents expressing an interest in the service who will eventually subscribe.

The analysis of the market research data based on a sample survey, where a stratified sample is drawn across market segments, for a service that is newly introduced or is planned, is discussed below:

Let

X_{1i} = the proportion of firms in market segment i | hat are very interested in the service.

X_{2i} = the proportion of firms in market segment i | hat are interested in the service.

X_{3i} = the proportion of firms in market segment i | hat are not interested in the service.

X_{4i} = the proportion of firms in market segment i | hat cannot decide whether they are interested or not.

The above example has 4 categories of responses. Greater of fewer categories may be used depending on the design of the questionnaire.

Notice that

where j = the index of categories of responses.

Market research firms sometimes determine conversion ratios for selected product/service types. Conversion ratios depend on the nature of the service, the type of respondents, and the questionnaire and its implementation. Conversion ratios applied to the sample will estimate the expected proportion of firms *in the survey* | hat will eventually subscribe, over the planning period. For studies related to the estimation of conversion ratios, refer to [1], [3] and [5].

Then,

c_1X_{1i} = the proportion of firms in market segment i | hat expressed a strong interest and are expected to subscribe.

c_2X_{2i} = the proportion of firms in market segment i | hat expressed an interest and are expected to subscribe.

c_3X_{3i} = the proportion of firms in market segment i | hat expressed no interest but are expected to subscribe.

c_4X_{4i} = the proportion of undecided firms in market segment i | hat are expected to subscribe.

where c_j = conversion ratio for response j .

The proportion of firms in market segment i , P_i, that are expected to subscribe to the service, equals

The conversion ratio is based on the assumption that there is a 100% market awareness. That is, all surveyees are fully informed of the service availability, use, tariffs, technical parameters, etc. P_i,

therefore, can be interpreted as the long-run proportion of firms in market segment i | hat are expected to subscribe to the service at some future time period, T .

Two issues arise in the estimation of the proportion of customers that subscribe to the service:

1) while P_i refers to the sample surveyed, the results need to be extrapolated to represent the population.

2) P_i is the long-run (maximum) proportion of firms expected to subscribe. We are interested in predicting no just the eventual number of subscribers but, also, those at intermediate time periods before the service reaches a saturation point.

5.4 *Extrapolation from sample to population*

To extrapolate the data from the sample to represent the population, let

N_i = size of market segment i | measured for example, by the number of firms in market segment i)

Then S_i, the expected number of subscribers in the planning horizon, equals:

$$S_i = P_i N_i \tag{5-2}$$

5.5 *Market penetration over time*

To determine the expected number of subscribers at various points in time before the service reaches maturity, let

$p_{i\backslash dt}$ = the proportion of firms in market segment i
| hat are expected to subscribe at time t .

Clearly,

$p_{i\backslash dt} < P_i$

and $p_{i\backslash dt}\ P_i$ as t | fIT

The relation between $p_{i\backslash dt}$ and P_i can be explicitly defined as:

(5-3) $$p_{i\backslash dt} = a_{i\backslash dt} \times P_i$$

$a_{i\backslash dt}$ is a penetration function, reflecting changing market awareness and acceptance of the service over time, in market segment i . An appropriate functional form for $a_{i\backslash dt}$ should be bounded in the interval (0,1).

As an example, let $a_{i\backslash dt}$ be a logistic function:

(5-4) $$a_{i\backslash dt} =$$
[Formula Deleted]

b_i 0 is the speed with which $p_{i\backslash dt}$ approaches P_i in market segment i , as illustrated in Figure 1/E.508.

For other examples of non-linear penetration functions, refer to the Annex A.

Figure 1/E.508, p.

The introduction of a new service will usually differ according to the market segment. The rate of penetration may be expressed as a function of time, and the speed of adjustment (b_i) may vary across segments. Lower absolute values of b_i, for the logistic function will imply faster rates of penetration.

While the form of the penetration function relating the rate of penetration to time is the same for all segments, the parameter b_i varies across segments, being greater in segments with a later introduction of the new service.

Let t_{0i} = time period of introduction of service in market segment i .

Then, $t - t_{0i}$ = time period elapsed since service was introduced in market segment i .

In the diagramatic illustration, of Figure 2/E.508, the service has achieved the same level of market penetration a_0, in t_C periods after its introduction in market C as it did in t_A periods after its introduction in market segment A . Later introductions may not necessarily lead to faster rates of penetration across segments. However, within the same market segment, across countries with similar characteristics, such an expectation is reasonable.

Figure 2/E.508, p.

5.6 *Growth of market segment over time*

The above discussion has accounted for gradual market penetration of the new service, by allowing $p_{i\backslash dt}$ to adjust to P_i over time. The same argument can be extended to the size of market segment i | ver time.

Let $n_{i\backslash dt}$ = size of market segment i at time t .

Then, the expected number of subscribers at time t | n market segment i , equals:

$$s_{i\backslash dt} = a_{i\backslash dt} \times p_{i\backslash dt} \times n_{i\backslash dt} \qquad (5\text{-}5)$$

and

$S_t = \sum_i s_{i_t}$ = expected number of subscribers across all market segments at time t .

5.7 *Quantities forecasted*

The above procedure forecasts the expected number of customers for a new service within a country. Other quantities of interest may include lines, minutes, messages, revenue, packets, kilobits, etc. The most straight forward

forecasting method for some of these quantities is to assume constant relationships such as:

expected access lines = (average access lines) × expected number of subscribers

expected minutes = (average use per line) × expected access lines

expected messages = expected minutes/(average length of conversation)

expected revenue = (average rate per minute) × expected minutes

The constants, appearing in parentheses, above, can be determined through 1) the process of market research, or 2) past trends in similar services.

5.8 *Forecasting with historical data: application analysis*

After a new service has been introduced, historical data can be analyzed to forecast demand for expanded availability to other countries. Development of a new service will follow trends based on applications, such as data transmission, travel reservations, intracompany communications, and supplier contact. Applications of a service vary widely and no single variable may be an adequate indicator of total demand.

The following procedure links demand to country characteristics for forecasting expanded availability of a new service to other countries.

Let $D = (D_i, D_2, | \ | \ | \ | \ | \ |, D_n)'$

represent a vector of country-specific annual demand for the service across n | ountries, where the service currently exists. Let C = matrix of m | haracteristics relating to each of the n | ountries that are reasonable explanatory variables of demand. The components of m | ould vary depending on the nature of the service and its application.

Some essential components of m | ould be the price of the service (or an index representing its price) and some proxy for market awareness. As discussed in earlier sections, market awareness is one of the key determinants of the rate of market penetration of the service. Reasonable proxies would be advertising expenditures and time (measured as $t^* = t - t_0$) where t^* would measure time elapsed since the service was first introduced at time t_0. Market

awareness can be characterized as some non-linear function of t^*, as presented in § 5.5. Other components of m | ay include socio-economic characteristics of the customers, market size and location of customers.

The model that is estimated is:

$$D = C\beta + u \tag{5-6}$$

where

C is a $(n \times m)$ matrix of country characteristics

D is a $(n \times 1)$ vector of demand

β is a $(m \times 1)$ vector of coefficients corresponding to each of the m | haracteristics

$u = (n \times 1)$ vector of error terms

The estimated regression is:

$$\hat{D} = C\hat{\beta} \tag{5-7}$$

Traditional methods of estimating regressions will be applied. Equation (5-7) can be used for predicting demand for any country where the service is being newly introduced, as long as elements of the matrix C | re available.

5.9 *Forecasting with limited information*

In the extreme case where no market research data is available (or is uneconomical given resource constraints), or country characteristics that affect demand are not easily available or quantifiable, other methods of forecasting need to be devised.

For example, to forecast the demand for a new international private line service using digital technology, the following elements should be taken into account in the development of reasonable estimates of the expected number of lines:

a) discussions with foreign telephone companies,

b) discussions with very large potential customers regarding their future needs,

c) service inquiries from customers,

d) customer letters of intent, and

e) any other similar qualitative information.

6 Forecast tests and adjustments

6.1 *General*

Forecast tests and adjustments are dependent on the methodology applied. For example, in the case of a market research based forecast, it is important to track the forecast of market size, awareness and rate of penetration over time and to adjust forecasts accordingly. However, for an application-based methodology, traditional tests and adjustments applicable to regression methods will be employed, as discussed below.

6.2 *Market research based analysis*

This section discusses adjustments to forecasts based on the methodology described in §§ 5.2 to 5.8. The methodology was based on quantification of responses from a sample survey.

The forecast was done in two parts:

a) extrapolating the sample to the population, using market size, N_i;

b) allowing for gradual market penetration (awareness), $a_{i\backslash dt}$ of the new service over time.

The values attributed to $n_{i,t}$ (which represents the size of market segment i at time t) and $a_{i,t}$ can be tracked over time and forecast adjustments made in the following manner:

a) As an example for $n_{i,t}$, the segments could be categorized as travel or financial services. The size of the segment would be the number of tourists, and the number of large banks. Historical data, where available, on these units of measurement can be used to forecast their sizes at any point of time in the future. Where history is not available, reasonable growth factors can be developed through subject matter experts and past experiences. The forecast of $n_{i,t}$ should be tracked against actual measured values and adjusted for large deviations.

b) For $a_{i,t}$, testing with only a few observations since the introduction of the service is more difficult.

Given that,

$$a_{i,t} = \frac{p_{i,t}}{P_i} \qquad (6\text{-}1)$$

and P_i is assumed fixed (in the long run), testing $a_{i,t}$ is equivalent to testing $p_{i,t}$. $p_{i,t}$ can be tracked by observing the proportion of respondents that actually subscribe to the service at time t . This assumes the need to track the same individuals who were originally in the survey, as is customary in a panel survey. Panel data is collected through sample surveys of cross-sections of the same individuals, over time. This method is commonly used for household socio-economic surveys. Having observed $p_{i,t}$ for a new period, values of $a_{i,t}$ can be plotted against time to study the nature of the penetration function, $a_{i,t}$, and the most appropriate functional form that fits the data should be chosen. At very early stages of service introduction, traditional functional forms for market penetration, such as a logistic function (as illustrated in the example in § 5.5), will be a reasonable form to assume. Other variations of the functional form depicting market penetration would be the Gompertz or Gauss growth curves. The restriction is that the penetration function should be bounded in the interval (0,1). See Annex A for an algebraic depiction of functional forms.

There are various statistical forms that may be chosen as representations for the penetration function. The appropriate functional form should be based on some theoretical based information such as the expected nature of penetration of the specific service over time.

Continuous tracking of $n_{i,t}$, $p_{i,t}$ and $a_{i,t}$ over time will enable adjustments to these values whenever necessary and enable greater confidence in the forecasts.

6.3 *Application based analysis*

The application based analysis is a regression based approach and traditional forecast tests for a regression model will apply. For instance, hypothesis tests on each of the explanatory variables included in the model will be necessary. Corrections may be needed for hetero-elasticity, serial correlation and multicollinearity, when suspect. The methodology for performing such tests are described in most econometrics text books. In particular, references [2] and [4] can be used as guidelines. Recommendation E.507 also discusses these corrections.

Adjustments need to be made for variables that should be included in the regression model but are not easily quantifiable. For example, market awareness that results from advertising and promotional campaigns plays an important role in the growth of a new service, but data on such expenditures or the associated awareness may not be readily available. Some international services are targeted towards international travelers, and fluctuations in exchange rates will be a determining factor. Such variables, while not impossible to measure, may be expensive to acquire. However, expectations of future trends in such variables can enable the forecaster to arrive at some reasonable estimates of their impact on demand. Unexpected occurrences such as political turmoil and natural disasters in particular countries will also necessitate post forecast adjustments based upon managerial judgement.

Another important adjustment that may be necessary is the expected competition from other carriers offering similar or substitutable services. Competitor prices, if available, may be used as explanatory variables within the model and allow the measurement of a cross-price impact. In most situations, it is difficult to obtain competitor prices. In such cases, other methods of calculating competitor market shares need to be developed.

Regardless of forecasting methodology, the final forecasts will have to be reviewed by management responsible for planning the service as well as by network engineers in order to assess the feasibility both from a planning implementation and from a technical point of view.

ANNEX A

(to Recommendation E.508)

Penetration functions (growth curves)

Some examples of non-linear penetration functions are illustrated below:

A.1 *Logistic curve*

$$a_{i\backslash dt} = \alpha \, / \, \{ \, + e^{D} \mathrm{lF261} \; t \, \}$$

(A-1)

For α = 1, the curve is bounded in the interval (0,1). Changing *b* will alter the steepness of the curve. The higher the value of *b* , the faster the rate of penetration. This curve is S-shaped and is symmetrical about its point of inflection, the latter being where;

$$\frac{{}^{2} f \, \mathrm{Ia}_{t} \, {}^{2} \mathrm{Ii}_{t}}{} = 0$$

(A-2)

A.2 *Gompertz curve*

$$a_{i\backslash dt} = \alpha \exp$$

[Formula Deleted]

(A-3)

As t ∞ $a_{i\backslash dt}$ α, the limiting growth.

Holding k = 1 and α = 1, higher values of *b* will imply slower rates of penetration. This curve is also S-shaped like the logistic curve, but is not symmetrical about its inflection point.

When t = 0, then $a_{i\backslash dt} = \alpha e^{D} \mathrm{lF261} \; b$, which is the initial rate of penetration.

A.3 *Gauss curve*

$$a_{i\backslash dt} = \alpha \left(1 - e^{D \, \mathit{lF}\,261 \; f \, Ibt^{2}} \right)$$

(A-4)

As t ∞, then a_{it} α

As t 0, then a_{it} 0.

Choosing α = 1, the curve is bounded in the interval (0,1).

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SECTION 3

DETERMINATION OF THE NUMBER OF CIRCUITS IN MANUAL OPERATION

Recommendation E.510

DETERMINATION OF THE NUMBER OF CIRCUITS IN MANUAL OPERATION

1 The quality of an international manual demand service should be defined as the percentage of call requests which, during the average busy hour (as defined later under § 3) cannot be satisfied immediately because no circuit is free in the relation considered.

By *call requests satisfied immediately* are meant those for which the call is established by the same operator who received the call, and within a period of two minutes from receipt of that call, whether the operator (when she does not immediately find a free circuit) continues observation of the group of circuits, or whether she makes several attempts in the course of this period.

Ultimately, it will be desirable to evolve a corresponding definition based on the *average speed* of establishing calls in the busy hour, i.e. the average time which elapses between the moment when the operator has completed the recording of the call request and the moment when the called subscriber is on the line, or the caller receives the advice *subscriber engaged* , *no reply* , etc. But for the moment, in the absence of information about the operating time in the European international service, such a definition cannot be established.

2 The number of circuits it is necessary to allocate to an international relation, in order to obtain a given grade of service, should be determined as a function of the *total holding time* of the group in the busy hour.

The total holding time is the product of the number of calls in the busy hour and a factor which is the sum of the average call duration and the average operating time

These durations will be obtained by means of a large number of observations made during the busy hours, by agreement between the Administrations concerned. If necessary, the particulars entered on the tickets could also serve to determine the average duration of the calls.

The average call duration will be obtained by dividing the total number of minutes of conversation recorded by the recorded number of effective calls.

The average operating time will be obtained by dividing the total number of minutes given to operating (including ineffective calls) by the number of effective calls recorded.

3 The number of calls in the busy hour will be determined from the average of returns taken during the busy hours on a certain number of busy days in the year.

Exceptionally busy days, such as those which occur around certain holidays, etc., will be eliminated from these returns. The Administrations concerned should plan, whenever possible, to put additional circuits into service for these days.

In principle, these returns will be taken during the working days of two consecutive weeks, or during ten consecutive working days. If the monthly traffic curve shows only small variations, they will be repeated twice a year only. They will be taken three or four times a year or more if there are material seasonal variations, so that the average established is in accordance with all the characteristic periods of traffic flow.

This Recommendation dates from the XIIIth Plenary Assembly of the CCIF (London, 1946) and has not been fundamentally revised since. It was studied under Question 13/II in the Study Period 1968-1972 and was found to be still valid.

4 The total occupied time thus determined should be increased by a certain amount determined by agreement between the Administrations concerned according to the statistics of traffic growth during earlier years, to take account of the probable growth in traffic and the fact that putting new circuits into service takes place some time after they are first found to be necessary.

5 The total holding time of the circuits thus obtained, in conjunction with a suitable table (see Table 1/E.510), will enable the required number of circuits to be ascertained.

6 In the international manual telephone service, the following Tables A and B should be used as a basis of minimum allocation:

Table A corresponds to about 30% of calls failing at the first attempt because of all circuits being engaged and to about 20% of the calls being deferred.

Table B, corresponding to about 7% of calls deferred, will be used whenever possible.

These tables do not take account of the fact that the possibility of using secondary routes permits, particularly for small groups, an increase in the permissible occupation time.

H.T. [T1.510]
TABLE 1/E.510
Capacity of circuit groups
(See Supplement No. 2 at the end of this fascicle)

Number of circuits	Table A		Table B	
	Percentage of circuit usage	{		
	Percentage of circuit usage	{		
1	65.0	39	—	—
2	76.7	92	46.6	56
3	83.3	150	56.7	102
4	86.7	208	63.3	152
5	88.6	266	68.3	205
6	90.0	324	72.0	259
7	91.0	382	74.5	313
8	91.7	440	76.5	367
9	92.2	498	78.0	421
10	92.6	556	79.2	475
11	93.0	614	80.1	529
12	93.4	672	81.0	583
13	93.6	730	81.7	637
14	93.9	788	82.3	691
15	94.1	846	82.8	745
16	94.2	904	83.2	799
17	94.3	962	83.6	853
18	94.4	1020	83.9	907
19	94.5	1078	84.2	961
20	94.6	1136	84.6	1015

Note — Tables A and B can be extended for groups comprising more than 20 circuits by using the values given for 20 circuits.

Tableau 1/E.510 [T1.510], p.16